# Resurrection principles

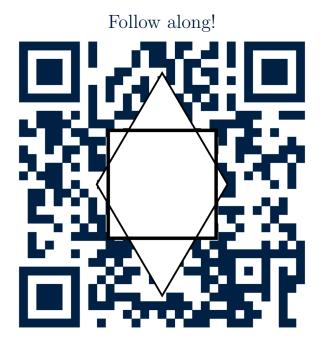


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# Maximality principle



The *Maximality principle* was introduced by Stavi and Väänänen, and independently Hamkins.





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so anything forceably necessary is already true.



### Some examples and non-examples



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" $S \subseteq \omega_1$  is non-stationary" is forceably necessary, because we can always shoot a club through its complement.



# Resurrection principle





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#### Example

Either CH or  $\neg \mathrm{CH}$  realizes an instance of the resurrection principle.



# Lightface and boldface variant



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The boldface variant, which we denote by  $\overrightarrow{RP}$ , talks about statements in the language of set theory with arbitrary real parameters (but no other parameters are allowed).

#### Observation

With real parameters in hand, one can refer to any hereditarily countable set. Specifically, a real x HC-codes  $y \in$  HC if and only if  $(\omega, \{ (n,m) \in \omega \times \omega \mid x(2^n 3^m) = 0 \}$  is isomorphic to  $(\text{TC}(\{y\}), \in)$ .



## The necessary boldface variant





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Consider any cardinal  $\kappa$  and its successor, in L and V. Let g be a V-generic real for  $\operatorname{Coll}(\omega, \kappa)$ . By  $\Box \operatorname{RP}_{\sim}$ , " $\omega_1^{L[g]}$  is countable" is true in V[g].



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In particular,  $0^{\#}$  exists.



# Qualified variants





A subtle point is that full resurrections— $\Box \operatorname{RP}$ ,  $\operatorname{RP}$ , and RP—are taken as schemes of assertions, but  $\Box \operatorname{RP}_{\Sigma_2}$  is a single axiom, since we have a universal  $\Sigma_2$  truth predicate, and can express it as one statement.



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This is the sense used in the theorem below.

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If there is a proper class of Woodin cardinals, then  $\Box\operatorname{RP}_{\Sigma_2}$  holds.



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A surprising corollary of Woodin's result is that (the existence of) I0 is resurrectable.



# Set-theoretic geology



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 $\mathbb{M} \models \mathbb{ZFC}.$ 

The mantle is a transitive, definable, and set forcing invariant. It is independent of ZFC whether the mantle is a ground or not.



# Ground Definability Theorem



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Suppose M is a ground of V and M[G] = V for some  $\mathbb{P} \in M$  and  $(M, \mathbb{P})$ -generic G. Then, M is  $\Sigma_2$ -definable in V.



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Corollary

If BA holds, then the mantle is  $\Sigma_2$ -definable in V.



# Usuba's extendible cardinal

A cardinal  $\kappa$  is *extendible* if for every ordinal  $\lambda$ , there is an ordinal  $\gamma > \lambda$  and an elementary embedding  $j: V_{\lambda} \to V_{\gamma}$  with a critical point equal to  $\kappa$ , which is to say that for any  $x \in V_{\lambda}$ , the cardinality of x is invariant under j just in case its less than  $\kappa$ .



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Theorem (Goldberg)

It is consistent that Usuba's theorem fails in  $V_{\kappa}$ , where  $\kappa$  is the least extendible cardinal.



# The failure of resurrection



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- Take a sufficiently large  $\lambda$ , so that  $(\lambda^+)^{\mathbb{M}} = \lambda^+$ .
- Let g be a V-generic real for  $\operatorname{Coll}(\omega, \lambda^+)$ .
- Take any strong limit  $\gamma > \lambda$  with  $V_{\gamma} \prec_{\Sigma_2} V$ .



•  $V_{\gamma}$  thinks that  $\mathbb{M} \cap V_{\gamma}$  is a ground. So by  $\Sigma_2$ -elementarity,  $\mathbb{M} \cap V_{\gamma} = \mathbb{M}^{V_{\gamma}}$ .



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- So the assertion expressing that there is a  $\gamma > \lambda$  such that  $V_{\gamma} \prec_{\Sigma_2} V$  and  $V_{\gamma} \models (\lambda^+)^{\mathbb{M}} = \lambda^+$  gives a failure of  $\Box \operatorname{RP}_{\Sigma_3}$ .



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#### Observation

The refuting assertion is provably equivalent to the assertion that for all  $\gamma > \lambda$ ,  $V_{\gamma} \prec_{\Sigma_2} V$  implies that  $V_{\gamma} \models (\lambda^+)^{\mathbb{M}} = \lambda^+$ .



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### Corollary

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# State of the art





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The full necessary boldface resurrection principle  $\Box \stackrel{\text{RP}}{\underset{\text{$\sim}}{\text{$\sim}$}}$  is consistent with a proper class of I0 cardinals.



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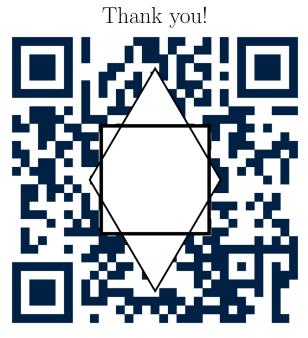
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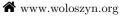
The full necessary boldface resurrection principle  $\square \stackrel{\frown}{\operatorname{RP}}$  is consistent with a proper class of I0 cardinals.

This,  $\Box RP_{\Pi_2}$ , and more, is a work in progress.









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